2017

A Real Options Approach to Criminal Careers

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Abstract: This paper proposes a dynamic model based on real options to evaluate the criminal career. In the model, individuals can choose the best moment to engage in crime (illegal activity). The model proposed allows the evaluation of the impact of different risk preferences, punishment probability, punishment severity and, mainly time discount in the individual's decision. Through model calibration it is possible to observe that the option for a criminal career depends on a high return in the illegal activity even when individuals are risk neutral and when they have a low time discount. The paper also discusses youth participation in crime.

Keywords: career, crime, real options.

I. INTRODUCTION

A criminal career is the longitudinal characterization of a sequence of crimes committed by an individual (Blumstein et al., 1986), in other words, it represents the criminal trajectory of individuals since their first to their last crime (Blumstein, Cohen & Hsieh, 1982). Although this definition is simple and easily understood, it is one of the most difficult research areas to deal with, within the economics of crime. Difficulties are both theoretical and empirical. Regarding empirical models, the difficulties go from the way of measuring a career, passes through the way of obtaining a sample and how to design an experiment (Piquero, Brame & Lynam, 2004), to how and when it ends (Laub & Sampson, 2001). On the other hand, theoretical models that fit the criminal career are necessarily dynamic and this brings along, of course, a difficulty in their mathematical treatment, since most of the models presented in the literature do not have an analytical solution.

It is still acceptable, in part of the literature, that a small group of individuals (chronic criminals) is responsible for most of the criminal activity (Visher, 1986; Piehl & Diiulio, 1995; Blumstein et al., 1982; Piquero et al., 2007). The behavior of these criminals, beyond being of great interest to the justice system, is the focus of this paper, which aims to study the criminal career, from a theoretical approach.

Traditional economics of crime models are insufficient to model a criminal career due to their static nature. Individuals make choices at a certain point in time, respond to exogenous incentives and, according to their preferences and restrictions, decisions are made (in the present) regarding future opportunities.

Moreover, the traditional way of modeling crime in economics is the portfolio choice. In these models, the representative individuals should allocate their leisure time, legal and illegal activities, taking into account the risks involved in the second type[1]. In this way, it is possible to obtain an optimal allocation for each activity. However, some illegal activities are only economically viable if there is a repetition of crimes, what in language of criminology is called recidivism.

There is an incentive to relapse since punishment might not include all the crimes practiced by the individual, because the criminal can be punished only for one or two crimes. Since the utility grows according to the amount of crimes

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[1] Block and Heinecke (1975) and Heineke (1978) are the precursors of the application to the economics of crime. Ehrlich (1973) presents a rudimentary version of choice between the legal and illegal markets; however the model does not involve time allocation.

committed and the cost of punishment grows at a lower rate, there is a welfare gain for the criminal when engaging in a criminal career, due to a form of scale gain presented by crime. Besides, the use of new protection technology by the potential victims increases the cost of practicing crimes and creates an entrance barrier to illegal activities. However, this cost can be compensated with the repetition of the crimes. These forms of incentives cannot be captured through static models.

Opting for a criminal career involves costs that might be paid for throughout the life cycle of an individual. In the case of punishment by the legal system, the stigma of punishment can significantly reduce this individual's income in the future (Lott, 1992). In the meantime, this cost will be paid only in the future and, therefore, the individual's intertemporal preferences will be relevant in order to determine the impact of this (future) cost on the decision made in the present. It has implications for the debate over the effect of severity of punishment on deterrence. Becker (1968) discusses the optimal combination of certainty and severity of punishment to deter crime. However, as argued by Mastrobuoni and Rivers (2016), Becker´s model predicts that doubling the sentence length leads to a doubling of the costs paid by criminals and that would be equivalent to a doubling of punishment probability. However, this is true only when criminals do not discount the future. Sentence lengths may have decreasing deterrence power over time when compared to punishment probability as sentences increases since they are paid in the future.

Through these arguments, dynamic effects can be relevant in explaining criminal's behavior, be it chronic or not, since effects of capital accumulation or intertemporal discount rate (Flinn, 1986; Mocan et al., 2000) or repeated crimes (Spelman, 1994; Piquero et al., 2007; McCrary, 2009) cannot be addressed by static models.

This paper proposes a dynamic theoretical model for the criminal career within the framework of real options. The model parts from some assumptions regarding the behavior of these criminals that are related to the characteristics of real options.

In the first place, chronic criminals usually do not participate in legal activities and tend to develop a career in illegal activities. Establishing a career in illegal activities merely reflects the stability of opportunities (incentives) that the criminal faces. For example, when committing a crime, a psychological cost, that reduces the recidivism cost, is overcome. Besides, an eventual conviction is capable of reducing the possible gain, in a legal activity, due to the stigma of being an ex-convict. These characteristics imply that it is possible to model crime considering the choice between one sector and the other as mutually exclusive events throughout the individual's lifetime. This results in irreversibility towards

the choice of illegal activities since, once the individual exercises this option, he will rarely go back to the legal activity market.

In the second place, decisions can be taken at any moment that will bear consequences for the rest of the individual's life cycle. In static models, the decision to engage in illegal activities is taken, at a certain moment in time, and there is no postponement option. Meanwhile, in fact, this engagement can be made at any time during the individual's life.

In the third place, the revenue from illegal activity is uncertain. The possibility of success or failure in each event (crime) allows us to suppose that the income of illegal activity is stochastic. Mocan et al. (2000) and McCrary (2009) associate the revenue of crime to a probability distribution. However, they do not establish any *a priori* distribution. This significantly complicates the resolution of the models and the interpretation of their results. This paper, as well as the traditional models of real options, proposes that the income of illegal activities follow a mixed process, partly continuous and partly discrete, that is to say, subjected to a geometric Brownian motion (GBM) with discrete jumps. In this way, they have a distribution similar to a log-normal, however with "heavier" tails. This allows us to obtain an analytical solution for the model and makes it different from others, since it presents a closed solution with less ambiguous results.

In spite of the model having identical characteristics to those of a financial option (American options style), the dimensions of the criminal career, addressed by criminologists, are taken into account[2]. Criminological literature highlights three basic dimensions: participation, frequency of crimes and the duration of the criminal career. Participation is modeled from the option that the individual has of engaging in the criminal activity at any point in time. This option, once exercised, generates a continuous income stream determined by the frequency of the crimes and by their rate of success. On the other hand, the duration of the career is included in the model as a Poisson process. This process is determined by the probability of receiving a conviction and its subsequent punishment by the legal system[3].

Besides this brief introduction, this paper presents three more sections; the next one revises the dynamic theoretical models of the criminal career. The third section presents the proposed theoretical model, as well as its results and

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[2] See Piquero et al. (2007) for a review of empirical works regarding these dimensions.

[3] The possibility of the individual dying in a confrontation with the victims, the police or other criminals should not be ignored. From the model´s point of view, the outcome is the same, since it results in the criminal´s career shutdown.

interpretation. The fourth section presents the conclusions, based on what has been shown in the previous sections.

II. DYNAMIC MODELS: LITERATURE REVIEW

Few works present dynamic models of crime based on the life cycle of the individuals. Flinn (1986) is one of the pioneers. The author presents three models of time allocation for legal and illegal (crime) work in each period. The first one is a model with constant legal work salaries, the second one with human capital accumulation in the legal sector and, the third model, with increasing penalties for criminal activity. His concern was to reproduce the empirical models that show that participation in crime varies with age (Blumstein et al., 1986; Elliott et al., 1989; Farrington, 1986; Piquero et al., 2003; Wolfgang et al., 1987; Piquero et al., 2007). Although the article does not present any calibration, the model proposed shows that time allocation in crime is higher among younger people, just as the empirical models predict.

Mocan et al. (2000) propose a state dependent model in which the accumulation of human capital, in the legal and illegal sectors, determines the income obtained in these sectors. In this way, the accumulation obtained in one of the sectors in the previous period implies higher yields in this same sector in the following period, leading to a behavior of inertia during the individual's life cycle of those who tend to remain in one of the sectors initially chosen[4]. A contribution of this work is to present the transition dynamics in the life cycle of the individuals, something which is also absent in other works regarding the criminal career. The main limitation of the work is not being able to obtain an analytical solution for the model due to the complexity of its formulation, thus, its results are interpreted only through the calibration of the model.

Lee and McCrary (2005) do not present a dynamic model of crime, but discuss some of the fundamental aspects for dynamic modeling in crime economics. The work contributes to the literature when it highlights that more serious crimes are punished with penalties that involve freedom deprivation for a long period of time and, therefore, the individual's horizon of time is relevant to the discussion regarding the deterrence effects of the legal system.

McCrary (2009) presents a dynamic model that is the generalization of the model proposed by Becker (1968). Differently from Mocan et al. (2000), the author finds an analytical solution for the model through a critical return of crime, represented in the model by a reservation income return, just like the traditional models of the labor market. The work also presents a model for crime demand,

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[4] According to the authors this would lead the individuals to remain in the illegal sector even during periods after economic recessions; periods in which they were out of the legal working sector.

based on the role of the government, regarding the allocation of resources in the justice system that determine the probability of apprehension and conviction and, consequently, the punishment of criminals. The results of the comparative statics of the model in relation to the parameters are consistent with the literature.

Engelen (2004) and At and Chappe (2005) propose an alternative crime model based on real options. According to the authors, dynamic models based on the maximization of the expected utility resemble the net present value calculation made in corporate finance and, therefore, possess all the limitations already recognized in the literature when dealing with uncertainty (Dixit & Pindyck, 1994). Although the model presented by Engelen (2004) and At and Chappe (2005) reach the same conclusions as the static models, the authors conclude that the deterrent effect of a crime is not necessarily permanent; in other words, the decision of not committing a crime may represent just a simple postponement of this decision.

Anyway, the investment under uncertainty literature shows that, in the presence of three characteristics, that is to say, uncertainty, irreversibility and freedom of choice regarding the moment in which to commit a crime, the use of modeling based on real options presents some advantages over traditional modeling. Issues such as intertemporal substitution rates, risk aversion, income volatility and the impact of uncertainty on the decision of engaging in crime are dealt with in a better way. For example, both the incomes of legal and illegal activities involve some uncertainty (risk), thus, the higher risk of criminal activity must imply a higher volatility of its income. On the other hand, this should also carry a higher income, because, if it were not so, the illegal activity would not be advantageous. Therefore, such as most of the assets in economy, there is a relation between risk and return.

Notwithstanding, the works by Engelen (2004) and At and Chappe (2005) leave some gaps. At and Chappe (2005) present a simple real options model in which individuals possess deterministic incomes where the only source of uncertainty is the probability distribution associated to punishment, therefore, they ignore the effects of income uncertainty when deciding to commit a crime.

On the other hand, Engelen (2004) recognizes the limitations of his work, according to the author, p. 344: "*However, it is more realistic to assume that criminals can exercise their criminal option during the whole time to maturity (American option type)*". When using a European option model, it is assumed that the option can only be exercised at its maturity; the author admits that this hypothesis is not realistic.

In modeling crime with the real options approach, more specifically as an American options type with dividends, the returns of criminal activity are

modeled as a stochastic process. The most common and easiest way of modeling a stochastic process is in its continuous form through a Wiener process (Brownian motion)[5].

Thus, the next section presents a model based on real options in order to assess the decision of following a criminal career, that is to say, to engage in an illegal activity for a long period until this activity ceases due to external causes, such as disability through imprisonment or death.

III. A DYNAMIC MODEL OF A CRIMINAL CAREER

The model presented in this paper can be considered an extension of Becker's (1968) and Ehrlich's (1973) models, since it parts from the same principles. In the model, the individuals can opt for an activity in the legal sector or for an activity in the illegal sector.

Although there is a diversity of activities classified as illegal these possess some common characteristics, such as, the possibility of pecuniary gain and the risk of a verdict subjected to punishment for a period of time (deprivation of some freedom) capable of generating both psychic losses (subjective) and monetary losses for the individuals. These monetary losses can be in the form of fines, in the form of opportunity cost (there is no income generation during the punishment period) or a form of future income loss in the legal sector, due to the period without human capital accumulation and to the stigma of having a criminal record. Moreover, as it has been commented previously, certain illegal activities incur death risk. For this reason it is necessary to attribute a stochastic character to illegal activity[6]. On the other hand, due to its simplicity, it is assumed that legal activity has deterministic revenues.

There is no need of training to participate in the legal and illegal sectors and there is no entrance and exit cost from these sectors, in spite of having an opportunity cost due to the option made. It is assumed that the participation in these sectors is mutually exclusive and that there is no return to a legal activity after a verdict.

The first hypothesis is justified by the interest of the present paper in studying the career of the chronic criminal and not that of occasional criminals and the second is based on the literature that shows that convictions derive in lower revenues and the impossibility of exercising a number of legal activities

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[5] The Brownian motion characteristics (of Markovian processes) are the ones that ensure the analytical solution of the real option models. See Trigeorgis (1996) for a presentation of the main models.

[6] In this aspect, there is a similarity with risky jobs, since the illegal activity can be understood as a death risk activity. See Rosen (1974) and Thaler and Rosen (1976) for a formalized study of these markets.

(Lott, 1992). Moreover, the stability of the model's parameters is assumed, in order to ensure the time consistency of the choices made.

Different from the traditional economics of crime models, the proposed model incorporates a dynamics to the individual choice and allows the choice to be made at any point in time of his life cycle. In this way, the engagement in illegal activity is an option that can be postponed and carried out at a convenient time[7]. Engelen (2004) argues that a crime may involve a low benefit at present and, that postponing its execution might generate some gain if there is an increase in these benefits, thereby, adding value to the option for an illegal activity. It should be pointed out that, according to the traditional theory of crime economics, this would not be possible because decisions are taken at an only instant *t*, since we are dealing with static models.

Individuals believe they will live for an infinite period of time and make decisions that involve consequences for a whole lifetime. The present value of the revenue obtained through legal activities represents the opportunity cost paid by criminals when exercising the choice for an illegal activity and can present two forms according to the horizon considered for the time of permanence in this sector:

- Infinite:

$$W = \int_0^\infty w_t e^{(\mu-\rho)t} dt = \frac{w_t}{\rho-\mu} \qquad [1]$$

- Finite:

$$W = \int_0^T w_t e^{(\mu-\rho)t} dt = \frac{w_t[1-e^{(\mu-\rho)T}]}{\rho-\mu} \qquad [2]$$

Where w_t represents the legal activity income at each instant *t*, this is exogenous in the model because it is determined in the labor market within which the illegal sector is only a small fraction (Ehrlich, 1996), ρ represents the intertemporal time discount of the individual and μ represents the increments (reductions) in w_t. The difference (ρ-μ) represents the yields associated to the assets (active), since it can be interpreted as a difference between the expected return and the asset valuation rate (McDonald and Siegel, 1986). In this way, the $\rho > \mu$ condition is necessary in order to obtain positive yields and, consequently, the revenue of the legal market will have a positive present value.

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[7] Although this is not common in economics of crime models, the optimal moment when to exercise the option (accepting a job offer) are studied through the "Job Search" model first introduced by McCall (1970). See Lippman and McCall (1976) for a review of these models.

On the other hand, the income obtained through illegal activity is subjected to uncertainty. In this paper, it is assumed that the monetary revenue of this type of activity is a stochastic process that follows a geometric Brownian motion (GBM), that is to say, it is assumed that the revenue percentage variations are normally distributed. A revenue that follows an GBM is a continuous representation of the limit of a binomial tree, with success (crimes with revenue) or failure (crimes without revenue), in which each increment is independent in relation to the others, that is to say, the probability of success during a certain period is independent from what has taken place during previous periods. Therefore, crime income is random once it depends of the success rate of crime execution.

Beyond the uncertainty regarding revenue, illegal activity is subjected to the uncertainty associated to a punishment originated in a conviction, the probability of which is associated to a Poisson process[8], in such a way that the probability of being punished and consequently the termination of the career, at any moment t is of λdT and that of continuing with the career is of $1-\lambda dT$. Then, the probability of practicing crimes for n periods until the termination of the career at $T=\tau$ is $1-e^{-\lambda\tau}$. After this period, the individual is penalized with a fraction $0\leq\phi\leq1$ of their return that represents the size of the punishment (severity).

This punishment is established by the legal system exogenously and depends upon the type of crime practiced[9]. In this way, the individual receives Y from the T moment in which he exercises the option for an illegal activity until the instant τ in which he is punished and receives $(1-\phi)Y$. Under these conditions the choice under uncertainty dynamic problem of the individual will be[10]:

$$Max \ \mathbb{E}\left[\int_0^\tau U(Y_t)e^{-\rho t}dt + \int_\tau^\infty (1-\phi)U(Y_t)e^{-\rho t}dt\right]$$

$$s.t. \quad \frac{dY}{Y} = \mu dt + \sigma dz - dq \tag{3}$$

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[8] This type of mixed model with continuous and discrete components can be seen originally in Merton (1976).

[9] In Merton (1976) and McDonald and Siegel (1986) these values are unknown. Here it is assumed that the individual knows the punishment imposed for the crime practiced. This supposition does not significantly change the results.

[10] Note that the aggregation made includes all the periods of activity and inactivity after exercising the option of following a criminal career. If there is no entrance and exit costs to these periods, it is indifferent whether these periods are alternate or continuous, since their aggregation generates the same sum (integral).

Where $\mathbb{E}$ represents the expectation operator, $U(.)$ represents the monetary utility function of the criminal activity and Y represents the monetary income of the criminal activity, parameter µ represents the positive or negative increments of the monetary income of the criminal activity[11] and σ represents the volatility of this increment, z represents a Wiener process in which $dz = \xi_t\sqrt{dt}$ and $\xi_t \sim N(0,1)$, q represent a Poisson process, such that dq is equal to $\varepsilon(\phi)$ with probability λdt and equal to zero with probability *1-λdt*. Where $\varepsilon(\phi)$ represents the impact of ϕ (severity of punishment) on the income of the illegal activity.

There are two sources of uncertainty in the model. The first one refers to the uncertainty regarding the profit of the illegal activity—that will depend of the rate of success in the activity—and the second one refers to the uncertainty regarding punishment.

The option of entering the criminal career is assessed considering the option value when this is exercised (value of being active) and the opportunity cost for not participating of the criminal activity (value of being idle). The value of being active is given by the optimum condition of the Hamilton-Jacobi-Bellman equation:

$$\rho F(Y)dt = \mathbb{E}(dF) + U(Y)dt \qquad\qquad [4]$$

Where $F(Y)$ represents the value of the option of joining the criminal career. This condition determines that the expected return must be equal to the capital gain (first term at the right) added to the instant flow of dividends (second term at the right). This last one depends of the utility function chosen. In this model, we opt for a von Neumann–Morgenstern exponential utility model, given by $U(Y) = Y^\theta$. Where the θ>0 exponent determines the individual's risk preference, if θ>1, is a risk preferer, if θ<1, is risk-averse and if θ=1, is risk neutral.

To obtain $\mathbb{E}(dF)$ the Itô's lemma is used and the expectation operator is applied. Substituting this expression and the utility function chosen in [4] we have that:

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[11] This parameter, once the option for the criminal activity is exercised, captures the gain income that the repetition of crimes might generate. In other words, this gain could, for example, originate in a learning process which not necessarily is internal and involves external issues, such as the "*learning by doing*" process, proposed by Arrow (1962).

$$\frac{\sigma^2}{2}Y^2 F''(Y) + \mu Y F'(Y) - (\rho + \lambda)F(Y) + \lambda F[Y(1-\phi)] + Y^\theta = 0 \qquad [5]$$

This is a non-homogeneous ordinary differential equation. Its solution is composed by a homogeneous solution and a particular one. The particular solution can be obtained using the indeterminate coefficients method. Conjecturing a solution with the following form:

$$F(Y)_P = c_1 Y^\theta \qquad [6]$$

Substituting this possible solution in [5] the following value is obtained for the constant:

$$c_1 = \frac{1}{\Delta'} \qquad [7]$$

Where $\quad \Delta' = \rho - \mu\theta - \frac{\sigma^2}{2}\theta(\theta - 1) + \lambda[1 - (1-\phi)^\theta].$ Thus, the particular solution will be:

$$F(Y)_P = \frac{U(Y)}{\Delta} \qquad [8]$$

The homogeneous solution can be obtained in the potencies form using Y^γ. This results in an indicial equation with two different real roots but with opposite signs, in other words, this equation can be written as follows:

$$F(Y)_H = k_1 Y^{\gamma_1} + k_2 Y^{-\gamma_2} \qquad [9]$$

Where k_1 and k_2 are constants to be determined. In the meantime, this problem only makes economic sense if the possibility of speculative bubbles occurring can be excluded. These are eliminated considering the non-overvaluation condition and the *F(0)=0* condition. The first one implies that it is not possible to obtain profit selling the option for a higher price than its fundamentals, therefore the $k_1=0$ condition is necessary, for this not to happen. The second one implies that $k_2=0$ is a necessary condition, once that the negative potential of Y goes to infinity when Y goes to zero. Under these conditions, the

value of being active will be only determined by the particular solution, in other words, by its fundamentals.

The value of being idle, in terms of criminal activity, does not render income, once the potential criminal is not exercising an illegal activity. Therefore, this is composed by the opportunity cost and by the expected option valuation. In this case, the Hamilton-Jacobi-Bellman equation will be:

$$\rho F_0(Y)dt = \mathbb{E}(dF) \tag{10}$$

Using the same procedures as before, the expression for the idle value will be:

$$\frac{\sigma^2}{2} Y^2 F_0''(Y) + \mu Y F_0'(Y) - (\rho + \lambda)F_0(Y) + \lambda F_0[Y(1 - \phi)] = 0 \tag{11}$$

This homogenous differential equation has the form of Cauchy-Euler and its solution can be obtained in the form of potencies. This indicial equation also has two different real roots with opposite signs, and so, such as in the previous problem, the negative root is eliminated by the same argument. Therefore, the solution will be given by:

$$F_0(Y) = k_1 Y^{\gamma_1} \tag{12}$$

Where k_1 is a constant to be determined and if $\phi=1$, we have $\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}\right]^{1/2} > 1$. In order to obtain this constant two boundary conditions are used:

$$F_0(Y^*) = F(Y^*) - W$$
$$F_0'(Y^*) = F'(Y^*) \tag{13}$$

Where Y^* represents the critical value of the monetary income of the criminal activity, which turns the criminal career into the best choice, in other words, it represents a reservation income of the criminal activity. The first boundary condition establishes that the value of the option must be the same as

the liquid monetary value obtained when this activity is practiced (value matching condition). The second condition establishes that these values must be tangent to the point in which it is an optimum choice to exercise the option (smooth-pasting condition). Using these conditions, a non-linear system is obtained, with two equations and two unknowns (k_1 and Y^*). This system has the following solutions:

$$Y^* = \left[\frac{\Delta' \gamma_1 W}{(\gamma_1 - \theta)} \right]^{\frac{1}{\theta}} \qquad\qquad [14]$$

$$k_1 = \frac{\theta}{\Delta' \gamma_1} \left(\frac{\Delta' \gamma_1 W}{\gamma_1 - \theta} \right)^{\theta - \gamma_1 / \theta} \qquad\qquad [15]$$

Where the condition $\gamma_1 > \theta$ is necessary to obtain Y^* and k_1 greater than zero. The result for Y^* implies that the present value of the monetary revenues of the illegal activity must be higher than the cost of the opportunity of acting in a legal activity. This can be seen substituting [14] and in [8], since in this way the liquid value of being active will be:

$$F(Y^*)_{liquid} = \left(\frac{\gamma_1}{\gamma_1 - \theta} \right) W \qquad\qquad [16]$$

Where the expression between brackets that multiplies W is higher than one, since $\gamma_1 > \theta > 0$. This result is the consequence of including the uncertainty regarding monetary revenues from the illegal activity in the model. It possible to show[12] that $\frac{\partial \gamma_1}{\partial \sigma} > 0$ e $\frac{\partial \gamma_1}{\partial \lambda} > 0$, in consequence, the income of the criminal activity must be increased to cover the opportunity cost of the legal activity, in other words, as Adam Smith predicted, risk subjected activities must include compensations in their revenues (Rosen, 1974; Thaler and Rosen, 1976). Accordingly, such as labor economic models, this model also finds a critical value Y^*, that represents the minimum revenue that an individual would be willing to receive in order to engage in an illegal activity. Furthermore, the result here presented is closed to all parameters and its interpretation is simpler than that of the dynamic models presented in the literature.

A. Comparative Static and Simulations

According to Becker (1968) individuals do not opt for illegal activities because they are different from other individuals in terms of basic motivations; they opt

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[12] For a detailed demonstration, see *Appendix B*.

for illegal activities because they possess different characteristics. These differences, in the model presented here, are represented by different parameters. In other words, different individuals possess different parameters and, therefore, make different decisions. The results obtained in the model can be better analyzed through comparative statistics and simulations.

The income reservation value obtained in [14] is related to the other model parameters in the following way; this will be greater when the likelihood of punishment is greater (λ) the severity of the punishment is greater (ϕ) the cost of opportunity for legal activity is greater (w). A reduction in the amount of reservation income occurs when the greater the coefficient of risk preference (θ), the larger the intertemporal time discount (ρ) and the higher the average income growth rate (μ).

The results are intuitive and follow the literature regarding the criminal's career. The way the model is built implies that W, the present revenue value in the legal market, represents the cost (opportunity) paid by the individual when exercising the option for an illegal activity. This is considered a single payment because its inclusion in the illegal sector implies its exclusion from the legal sector. In the model, this is an exogenous cost and there is no reference as to how this value is obtained. Yet, this value is defined in the labor market and depends of idiosyncratic characteristics such as the human capital accumulated by the individual, and depends of general economic characteristics such as a demand for workers in the economy's legal sector.

There is a vast literature that relates unemployment, in other words, opportunity cost equal to zero, to crime. The idea is that the lack of opportunity in the legal sector takes the opportunity cost to very low values and therefore, makes the option for the illegal activity more attractive (Eide et al., 2006). Chiricos (1987) using 288 estimations with aggregated data of 63 empiric works about crime determinants found that unemployment presents a positive signal and statistically significant coefficients in 31% of cases and only in 2% this signal was negative and the coefficient statistically significant. Similar results are found in Freeman (1995) and Levitt (1995). Therefore, the positive sign found for the derivative is confirmed by the empirical literature.

Ehrlich (1996) points out that general incentives such as salaries in the legal market hold a global effect on criminality; while, the impact of specific incentives, which are represented by the other parameters, cannot be ignored. An essential aspect of the model is to assess how the perspective of a revenue flow in the illegal sector can affect the decision of exercising the option for this sector. The horizon of this flow is directly related to the duration of the criminal's career which, in turn, depends of the subjective probability of punishment represented in

the model by λ^{13}. To better visualize the impact of the length of a criminal's career, let's consider a simulation with $\rho = 0.04, \mu = 0.02, \sigma = 0.1, \theta = 1, w = 1000$ and $t = \infty$. To obtain the reservation income Y^* it is necessary to use numerical methods to reach γ_1 when $0<\phi<1$. The results are summarized in Table 1.

TABLE 1

RESERVATION INCOME ($ PER YEAR) AS A FUNCTION OF λ AND ϕ

1/λ	ϕ			
	1	0.75	0.5	0.25
35	4052.45	3436.64	2778.45	2109.87
20	5250.00	4300.44	3308.54	2302.08
15	6172.40	4977.35	3742.82	2488.85
14	6434.77	5170.79	3868.36	2545.05
13	6736.92	5393.93	4013.69	2610.98
12	7088.70	5654.10	4183.73	2689.09
11	7503.49	5961.34	4385.16	2782.72
10	8000.00	6329.63	4627.33	2896.53
9	8605.17	6779.15	4923.67	3037.27
8	9359.29	7340.06	5294.30	3215.01
7	10325.53	8059.68	5770.71	3445.56
6	11608.78	9016.61	6405.24	3755.17
5	13397.18	10351.88	7291.74	4190.95
4	16065.52	12346.45	8617.21	4846.73
3	20484.22	15652.98	10816.13	5940.33
2	29250.00	22218.74	15185.55	8121.28
1	55250.00	41708.33	28166.51	14610.58

Table 1 shows that the increase in punishment and of the probability of being caught increase the necessary income for exercising the option for the illegal activity. For example, a risk neutral individual with an opportunity of earning $1000 per year forever in a legal activity would be willing to engage in an illegal activity if he received $ 4052.45 per year during 35 years. However, the

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[13] It should be noted that this may vary among individuals, despite being the same crime and justice system, since this depends also of the individual's capacity in hoodwinking the police and of defending himself in a law-court, for example, hiring good lawyers.

literature shows that the criminal career is not so long. The first works written by Greenberg (1975) and Shinnar and Shinnar (1975) estimate a career with duration between 5 and 12 years. Blumstein et al. (1982) find that most careers last, at least, 5 years and Spelman (1994) estimates an average period of 6 to 7 years. However, Piquero et al. (2004), using a longitudinal sample, find an average period of 17 years while Laub and Sampson (2003), using a similar sample, but with a different methodology, estimate the average career in 25 years.

In the model, long careers are directly associated to low probabilities of punishment. Kyvsgaard (2003) shows that these may vary quite a lot over crimes and over countries. For example, the probability of being punished[14] for a robbery would be of 11% in the United States, 17% in England, while in Denmark this probability would be of 33%.

Regarding other crimes, the probability would be lower still. The author estimates that the chance of being punished for the robbery of a car is 3% in the United States and 8% in Denmark and in England.

It should be pointed out that the duration of the career has implications regarding punishment policies, since they are directly related to the efficiency of incapacitation. Long imprisonment penalties for individuals with a small residual crime career are a waste of resources. Besides, the maintenance of older people in prison can lead to an increase of cost due to ill health of the prisoner (Piquero et. al., 2003).

Also regarding punishment, it is worth considering in the model at least two states of the same. When $\phi<1$ this means the punishment does not exhaust the capacity of generating resources in the illegal activity[15]. This would be the case of alternative penalties such as community service or semi-open prison regimes. Since these penalties involve a significant time allocation by the individual, he would have a partial reduction of his income. When $\phi=1$, then the individual will not obtain any income during the period of imprisonment.

Independent of the efficiency of punishment systems, the model replicates the existence of a "deterrence" effect both on the probability of punishment as well as the severity, already established in the literature[16].

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[14] This is obtained by the ratio between the number of convictions and the number of crimes per 1000 inhabitants.

[15] If there is no punishment ($\phi=0$) the reservation income considers only a source of uncertainty. In this case, following the parameters presented in Table 1, the reservation income would be $2425.39.

[16] It should be noted that many crimes are no longer practiced simply because they are not profitable. Therefore, it is possible to inhibit a crime without catching the criminal in flagrant before being committed. This is the essence of the deterrence concept.

Criminologists normally believe that the first is the most efficient in terms of criminality reduction than the second and economists, on the other hand, tend to believe there is a kind of trade off and that it is possible to find an optimal combination between them that may be more efficient[17].

Engelen (2004) believes that the deterrent effect might be only transitory since the option for crime can appreciate and make its exercise optimal at some period in the future. In the meantime, the dynamics of the model allow for the existence of a theoretical form of dissuading the individual to engage in the criminal career definitely. This result is presented as a proposition.

Proposition 1: *If Y*> Y(0), the optimal deterrence effect is $\lambda\phi = \mu$ for all $\mu > 0, 0 \leq \lambda \leq 1$ and $0 \leq \phi \leq 1$.*

Proof: *Appendix D.*

As the expected revenue from the illegal sector grows at a rate of $\mu - \lambda\phi$, if the expected penalty grows at the same rate as the revenue, there will not be an appreciation of the option. In this way, the income from the illegal sector will not exceed the reservation income, and the option for crime will not be exercised. Some authors argue that the difference $\mu - \lambda\phi$ must be constant for the activity to be viable and for the option to be exercised at some moment in time (Merton, 1976; McDonald & Siegel, 1986; Dixit & Pyndick, 1994). Thus, increases in the probability and in the severity of the punishment would be compensated with an increase in the income of the illegal sector, in other words, there would be an adjustment in the demand that would guarantee the existence of the illegal activity.

An implication of this result is that an increase in μ implies an increase in ρ since it is assumed that the assets generate positive yields. Thus, crimes with higher punishment probability (more frequent jumps) or with a harsher punishment (more intense jumps) tend to be practiced by individuals with a high intertemporal time discount rate, that is to say, individuals who have little regard for the future.

In the model proposed in this paper, as well as in the model proposed by Becker (1968), where potential criminals do not possess income restrictions, all the combinations of punishment probability and severity are capable of producing the same level of deterrence if the individuals are risk neutral. Nevertheless, the

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[17] See Polinsky and Shavell (2000) for a review.

impacts may have different magnitudes when considering individuals who are not risk neutral.

Proposition 2: *the elasticity of Y* regarding the probability of punishment is higher than the severity of the punishment if* $\frac{U(Y)-U(Y-f)}{f} > U'(Y-f)$, *in other words, the individuals are risk preferers.*

Proof: *Appendix E.*

Proposition 2, as well as Becker (1968), shows that, in order to maintain the same level of deterrence (same *Y* value*) the punishment probabilities and their severity must change according to the individual's risk preference. Individuals who are risk averse are more sensitive to changes in the probability of punishment than severity. Authors such as Witte (1980), Grogger (1991) and Block and Gerety (1995) conclude, through empirical studies, that criminals must risk preferers since Proposition 2 is verified such as proposed by Becker (1968). It occurs because, in the model, the illegal activity is a form of bet where individuals who appreciate risk, would show their preference for this option.

Eide (1994) finds an average value for the elasticity to the probability of punishment higher than the elasticity of the severity of punishment when collecting information from 118 estimations presented in the literature. This reinforces the argument that criminals are, in average, risk preferers. Furthermore, the literature shows that the preference of individuals regarding risk can be different for gains and losses and can also depend of the chance of gains and losses[18]. The preference for risk in losses and the aversion to risk in gains can be enough to find elasticity regarding the probability of punishment higher than the elasticity regarding the severity of the punishment (Foreman-Peck & Moore, 2010). Since punishment is a loss and crime revenue is a gain, an increase in the severity of the punishment will have a small effect on individuals with a propensity to risk, while an increase in the probability of punishment will substantially reduce the expected utility of an individual who is risk averse to gains.

The model allows assessing how the reservation income is altered with changes in the risk aversion parameters and income volatility of illegal activity.

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[18] The advances in behavioral studies regarding decisions show that individuals who are risk averse to gains with high probabilities and losses with low probabilities and appreciate risk for gain with low probability and losses with high probability (Tversky & Kahneman, 1992).

Table 2 presents the results considering a simulation with $\rho = 0.04, \mu = 0.02,$ $\lambda = 0.05, \phi = 1, w = 1000$ and $t = \infty$.

It is possible to observe that the values are quite sensitive to changes in the parameter that determines the risk preference. A risk averse individual, with θ=0.9 would be willing to exert the option for the illegal activity only if this activity presents a revenue equal to or 13 times as much as that which could be obtained in the legal market, while individuals who appreciate risk with θ=1.3 would exercise this option for a value 30% lower than the income they would obtain in a legal activity when σ=0.1.

TABLE 2

RESERVATION INCOME ($ P/YEAR) AS A FUNCTION OF θ AND σ

θ	σ				
	0.05	0.1	0.15	0.2	0.25
0.95	12068.71	13383.42	15041.17	16973.88	19176.29
0.91	10890.54	12075.67	13568.38	15306.54	17284.78
0.92	9849.35	10920.07	12267.16	13833.88	15614.77
0.93	8926.97	9896.42	11114.74	12529.99	14136.72
0.94	8107.91	8987.50	10091.68	11372.79	12825.46
0.95	7378.94	8178.62	9181.38	10343.43	11659.50
0.96	6728.70	7457.16	8369.60	9425.75	10620.42
0.97	6147.44	6812.29	7644.13	8605.86	9692.41
0.98	5626.75	6234.66	6994.44	7871.81	8861.87
0.99	5159.38	5716.23	6411.42	7213.27	8117.05
1	4739.05	5250.00	5887.21	6621.32	7447.77
1.01	4360.28	4829.92	5414.96	6088.20	6845.23
1.02	4018.35	4450.71	4988.75	5607.17	6301.76
1.03	3709.11	4107.79	4603.37	5172.35	5810.67
1.04	3428.94	3797.13	4254.32	4778.62	5366.14
1.05	3174.68	3515.22	3937.63	4421.48	4963.06

Empirical behavioral works show that only a fraction of the population is risk preferer to gains. Cramer et al. (2002) estimated that 1.39% of workers and 2.58% of employers appreciate risk. Diaz-Serrano and O'Neill (2004) found 6.5% in 1995 and 0.85% in the year 2000. Dohmen et al. (2005) identified 9% of their

sample as risk preferers. In the meantime, risk averse individuals might engage in criminal activity if this were sufficiently profitable or enough to cover their reservation income or if this risk preference parameter is simply associated to other parameters that reduce this critical value, such as low payment in the legal market and/or a higher intertemporal time discount.

In general, criminals have higher time discount rates (Wilson & Herrnstein, 1985; Katz et al., 2003). In an extreme case, in which criminals have infinite time discounts, punishments, such as imprisonment for a period of time, would have no effect. In fact, in this case, the relevant parameters are the individual preferences regarding risk and their income from the legal market.

Proposition 3: *When $\rho \to \infty$ the critical value $Y^* \to w^{1/\theta}$ independent of other parameters.*

Proof: *Appendix F.*

Proposition 3 shows that an infinite intertemporal time discount turns the individual indifferent between choosing a legal or an illegal activity, if he is risk neutral. As the revenue from legal activity is deterministic, the role giving to risk preference means only compensating the individual for the fact of having chosen the activity that involves risk. Therefore, a slightly superior income to that obtained in a legal activity is enough to make the option for the illegal activity optimal. Although there are few works that conclude that criminals might be risk averse, there is some empirical evidence of this. Shepherd (2003) shows empirical evidence in which criminals can be as risk averse as any law abiding citizen. The author shows there is a stigma associated to a conviction and this generates an increase in the expected penalty. In this way, individuals who can opt between certain and uncertain sentences (lottery), with the same expected sentence, would opt for the second due to the higher expected penalty associated to the first case.

Another, more theoretical line, shows that it is possible to have risk averse criminals through state-dependent or rank-dependent utility functions (Neilson & Winter, 1997), However, these are atypical utility functions that do not possess a reality based support. The model presented in this paper shows that it is possible for risk averse individuals to opt for a criminal career. Nevertheless, for risk averse individuals, the benefit of the illegal activity should be always higher to that of the revenue of a legal activity. Even in less extreme cases than those shown in Proposition 3, that is to say, with finite time discount, there is the possibility that risk averse individuals may opt for the criminal career, as long as

the demand for crime is able to generate enough income for this option to be optimal.

Table 3 shows how the reservation income is altered, with changes in the intertemporal time discount and during the period in which the individual obtains an income in the legal sector, considering a simulation with $\theta=1$, $\mu=0.02$, $\sigma=0.1$, $\lambda=0.05$, $\phi=1$ and $w=1000$.

Table 3. Reservation income (\$ per year) as a function of ρ and τ

ρ	τ				
	∞	35	30	25	20
0.02	8637.46	2550.74	2238.67	1910.60	1565.71
0.03	4882.78	2458.06	2203.05	1921.23	1609.76
0.04	3628.67	2358.86	2153.36	1914.61	1637.21
0.05	3000.00	2260.21	2096.42	1896.36	1652.01
0.06	2621.70	2166.12	2036.72	1870.57	1657.23
0.07	2368.70	2078.64	1977.16	1840.17	1655.26
0.08	2187.39	1998.63	1919.53	1807.28	1647.99
0.09	2050.94	1926.22	1864.88	1773.38	1636.86
0.1	1944.44	1861.12	1813.77	1739.50	1623.03
∞	1000.00	1000.00	1000.00	1000.00	1000.00

Just as there are preferences regarding risk, preferences regarding time also present heterogeneity related to circumstance. The current practice in economics is to assume the time discount constant for an individual, but this might differ between individuals (Becker, 1996). For example, Kirby et al. (1999) show that individuals who are drug users have their reasoning faculties affected and hold a much higher discount rate compared to what is assumed in economic models.

In the proposed model, the exponential intertemporal time discount does not allow assessment of the impact of this change on the same individual, although, in the meanwhile, it is possible to analyze different individuals at different points of their life cycle. For example, an individual who has a perspective of performing a legal activity during 35 years, receiving \$ 1000, and discounts future revenue at a 2% rate, would be willing to exert the option for the illegal activity if it paid at least \$ 2550.74 each period. However, if the discount rate were 10%, this amount would be reduced to \$ 1861.12 each period.

Changes in the individual's preference regarding risk and time are relevant explanations for a higher participation of youth in crime. There is empirical evidence that there is a positive relation between the individual's age and risk aversion (Jianakoplos & Bernasek, 1998; Palsson, 1996) and a negative relationship regarding intertemporal time discount (Kirby, 1997; Kirby & Marakovic, 1995; Myerson & Green, 1995).

Even though, there are also some authors who attribute the low revenues obtained in the legal labor market as the cause for this increased participation. Grogger (1998) maintains that criminal behavior responds to wages, and then the age distribution of crime may well be a labor market phenomenon. Besides, wages represent the opportunity cost of committing crime, and they rise steeply with age during the early part of one's career. Meanwhile, Witte and Witt (2000) point out that the evolution of revenue in legal activities cannot be the only explanation for a higher participation of young people in illegal activities, since many youths commit crimes way before having job opportunities in the legal sector. Lee and McCrary (2005) highlight the change in the penal regime for individuals who are over 18 years old. According to the authors, older age is responsible for a considerable increase in crime cost because they jointly increase the likelihood and severity of the punishments.

In short, it is possible to conclude that youngsters practice more crimes due to a set of factors, since they have a higher intertemporal time discount rate, lower risk aversion, less severe and less likely punishment and few chances in the legal work market that translate into a lower opportunity cost. When these characteristics are placed in the model proposed, they lead to a reduction in the reservation income of individuals who, in turn, would be willing to engage in a criminal career for a lower income. In fact, young people receive opportunities in the illegal activity because they represent a lower manpower cost than an adult.

IV. CONCLUSIONS

This paper presented a dynamic model of crime. Several components of crime and crime control are dynamic in nature and they cannot be captured by static models (McCrary, 2009). The results obtained are consistent with the literature of economics of crime. Besides, the model proposed presents some advantages in relation to previous ones. In the first place, it presents a closed analytical solution that avoids ambiguity. In the second place, it allows the simulation of numerical results from a theoretical model. The model also presents some advantages in relation to the model proposed by Engelen (2004). The use of a Poisson process, to model the punishment probability and its consequences on the revenues of the illegal sector, brings the model nearer to the real world in which there are at least two sources of uncertainty to these revenues. A further distinction to the author's work is that it uses an American model which is more realistic than European

models, since individuals do not have career options with an expiration date. In the real world, the option for the criminal career can be made at any moment during an individual's lifetime. However, it is necessary to point out that the model proposed has some limitations. The model presented here only models crime supply and, therefore, does not assess the costs of changes in the model's parameters, which could be modeled on the demand for crimes.

Therefore, it was not possible to infer as to optimal punishment policies, which are a relevant issue with a vast literature. The conclusions here presented, therefore, are limited to the offer of crimes, as well as the other dynamic models found in the literature, such as Mocan et al. (2000) and McCrary (2009). This limitation in the models of crime economics has already been highlighted by Merlo (2001), who emphasizes the importance of using a dynamic general equilibrium in order to model criminal behavior. This is a limitation that must be overcome in new works.

Finally, this paper leaves other gaps that can be approached in future works. It would be interesting to consider state-dependent revenues to individuals through the formalization of human capital accumulation and its implications for the insertion in the work market of both sectors. Another possibility is to consider the alternation and the concomitance between the sectors. This is a characteristic that would fit the behavior of sporadic criminals who would not fit the definition of a chronic criminal. In addition, dynamic models that have different forms for the intertemporal time discount could also generate interesting results. There is a robust literature indicating that exponential time discount rates are unsatisfactory in order to model the dynamic behavior of individuals.

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APPENDIX

A. dY moments

$$E(dY) = \left(\mu Y dt + \sigma Y \varepsilon \sqrt{dt}\right)\left(\frac{1-\lambda dt}{2}\right) + \left(\mu Y dt - \sigma Y \varepsilon \sqrt{dt}\right)\left(\frac{1-\lambda dt}{2}\right)$$
$$- \phi Y \lambda dt$$
$$= \mu Y dt(1 - \lambda dt) - \phi Y \lambda dt$$
$$= \underbrace{(\mu - \phi\lambda)}_{\text{expected rate of change in } Y} Y dt \qquad [\text{A.1}]$$

$$E(dY^2) = E(\mu^2 Y^2 dt^2 + \sigma^2 Y^2 dz^2 + (-Y)^2 dq^2) = \sigma^2 Y^2 dt + Y^2 \lambda \phi^2 dt$$

Where $E(dz^2) = dt$ and $E(dq^2) = \lambda\phi^2 dt$ and all terms with dt^2 are ignored, therefore, the variance of the process will be:

$$Var(dY) = E(dY^2) - [E(dY)]^2$$
$$Var(dY) = \underbrace{\lambda\phi^2 Y^2 dt}_{Poisson} + \underbrace{\sigma^2 Y^2 dt}_{GBM} \qquad [\text{A.2}]$$

B. Obtaining γ_1 and its derivatives

After substituting [13] in (12) and performing some algebraic manipulation, we have that

$$\frac{\gamma_1^2}{2} + \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)\gamma_1 - (\rho + \lambda) + \lambda(1-\phi)^{\gamma_1} \qquad [\text{A.3}]$$

Where $0 < \phi < 1$, the roots of this polynomial can be obtained through numerical methods such as Newton's method. However, when $\phi = 0$ and $\phi = 1$, these may be obtained analytically, and are, respectively:

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}\right]^{1/2} > 1 \iff \rho > \mu \qquad [A.4]$$

$$\gamma_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}\right]^{1/2} > 1 \iff \rho > \mu - \lambda \qquad [A.5]$$

Their derivatives with respect to parameters σ^2 and λ when $\phi = 1$ are, respectively:

$$\frac{\partial \gamma_1}{\partial \sigma^2} = \frac{2}{\sigma^3}\left\{\mu - \frac{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)(\mu+\rho+\lambda)}{\left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}\right]^{1/2}}\right\} < 0 \qquad [A.6]$$

$$\frac{\partial \gamma_1}{\partial \lambda} = -\frac{1}{\sigma^2\left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}\right]^{1/2}} < 0 \qquad [A.7]$$

C. Comparative statics of Y*

$$\frac{\partial Y^*}{\partial \lambda} = \frac{[1-(1-\phi)^\theta]}{\theta}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} > 0 \qquad [A.8]$$

$$\frac{\partial Y^*}{\partial \phi} = \lambda(1-\phi)^{\theta-1}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} > 0 \qquad [A.9]$$

$$\frac{\partial Y^*}{\partial \rho} = \frac{\gamma_i w + \Delta'\left[(\gamma_1-\theta)\gamma_1\frac{\partial W}{\partial \rho} + W\frac{\partial \gamma_1}{\partial \rho}\right]}{\theta(\gamma_1-\theta)^2}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} < 0 \qquad [A.10]$$

$$\frac{\partial Y^*}{\partial \mu} = \frac{-\theta w\left[(\gamma_1-\theta)\gamma_1 + \Delta\frac{\partial \gamma_1}{\partial \mu}\right]}{\theta(\gamma_1-\theta)^2}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} < 0 \qquad [A.11]$$

$$\frac{\partial Y^*}{\partial \sigma} = \frac{w\left[(\gamma_1-\theta)\gamma_1\frac{\partial \Delta'}{\partial \sigma} - \theta\Delta\frac{\partial \gamma_1}{\partial \sigma}\right]}{\theta(\gamma_1-\theta)^2}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} > 0 \qquad [A.12]$$

$$\frac{\partial Y^*}{\partial \theta} = \frac{\gamma_1 w\left[(\gamma_1-\theta)\frac{\partial \Delta'}{\partial \theta} + \Delta'\right]}{\theta(\gamma_1-\theta)^2}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1-\theta/\theta} < 0 \qquad [A.13]$$

$$\frac{\partial Y^*}{\partial w} = \frac{1}{\theta w}\left(\frac{\Delta' \gamma_i w}{\gamma_1-\theta}\right)^{1/\theta} > 0 \qquad [A.14]$$

D. Proof of Proposition 1

From [A.1] the following differential equation can be obtained:

$$\frac{\dot{Y}}{Y} = \mu - \lambda\phi \qquad\qquad\qquad [A.15]$$

Its solution is given by:

$$Y_t = Y_0 e^{(\mu-\lambda\phi)t} \qquad\qquad\qquad [A.16]$$

Substituting [15] in [A.16] and rearranging the terms, we have that

$$t = \frac{lnY_t^* - lnY_0}{\mu - \lambda\phi} \qquad\qquad\qquad [A.17]$$

Knowing that the optimal time to exercise the option is when $Y_t \geq Y^*$ and that if $Y^* > Y_0$ the option is not exercised in $t = 0$ (the numerator is a positive value), then, we have that $t \to \infty$ when $\lambda\phi \to \mu$.

E. Proof of proposition 2.

Beginning by Becker's model (1968), the expected utility of committing a crime is given by[19]:

$$E[U] = pU(Y - f) + (1 - p)U(Y) \qquad\qquad\qquad [A.18]$$

Where *U[.]* is the utility function, *Y* is the income from crime, both monetary plus psychic, *f* is the monetary equivalent of the punishment and *p* is the probability of being caught and convicted. In this way, an individual commits a crime if the expected utility is positive and does not commit it if it is negative. Note that:

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[19] Becker (1968), p. 177.

$$\frac{\partial E[U]}{\partial p} = U(Y - f) - U(Y) < 0 \quad \text{and} \quad \frac{\partial E[U]}{\partial f} = -pU'(Y - f) < 0 \qquad [\text{A.19}]$$

Using equation (A18) it is possible to construct the elasticities with respect to two parameters. These are given by:

$$\frac{-\partial E[U]}{\partial p}\frac{p}{U} = U(Y - f) - U(Y)\frac{p}{U} \quad \text{and} \quad \frac{-\partial E[U]}{\partial f}\frac{f}{U} = -pU'(Y - f)\frac{f}{U} \qquad [\text{A.20}]$$

Therefore, elasticity in relation to the probability of punishment will be greater regarding punishment if:

$$\frac{U(Y) - U(Y - f)}{f} > U'(Y - f) \qquad [\text{A.21}]$$

This occurs if $U''(Y\text{-}f)>0$, that is to say, when individuals are risk preferers.

In the model, the elasticity in relation to the probability of punishment $(E_{Y\lambda})$ will be greater than with respect to punishment $(E_{Y\phi})$ if $\frac{\lambda}{Y^*}\frac{\partial Y^*}{\partial \lambda} > \frac{\phi}{Y^*}\frac{\partial Y^*}{\partial \phi}$.

Using $\frac{\partial Y^*}{\partial \lambda}$ and $\frac{\partial Y^*}{\partial \phi}$ from *Appendix C* the inequality after removing the terms in common will be:

$$1 - (1 - \phi)^\theta > \phi\theta(1 - \phi)^{\theta-1} \qquad [\text{A.22}]$$

Denoting $\phi = \frac{f}{Y}$ and $1 - \phi = \frac{Y-f}{Y}$ and substituting in [7] we have that:

$$\frac{Y^\theta - (Y-f)^\theta}{f} > \theta(Y - f)^{\theta-1} \qquad [\text{A.23}]$$

Using that $U(Y) = Y^\theta$ equations [A.21] and [A.23] are equivalent.

F. Proof of proposition 3.

When $\rho \to \infty$, $\Delta' \to \infty$ and $\gamma_1 \to \infty$. Thus, substituting [1] in [15] we have that:

$$Y^* = \Lambda w_t^{\frac{1}{\theta}} \qquad\qquad\qquad [\text{A.24}]$$

Where $\Lambda = \left[\frac{\Delta'\gamma_1}{(\gamma_1-\theta)(\rho-\mu)}\right]^{\frac{1}{\theta}}$. Therefore, if $\rho \to \infty \Rightarrow \Lambda \to 1$, then, $Y^* \to w_t^{\frac{1}{\theta}}$.